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ON THE INTERACTION BETWEEN A PARTICLE AND A SPHERICAL BUBBLE AT LOW STOKES NUMBERS

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The motion of a small particle in a steady stream of liquid, uniform at infinity, is considered. Solution of equations of motion corresponding to a uniform and rectilinear immersion at infinity is obtained in the form of series in powers of Stokes numbers. The solution is used for defining the single act of flotation when the Stokes number is fairly low and the effect of inertial factors on the particle motion is insignificant. The first two approximations of the critical impact parameter with respect to the Stokes number, which define the probability of collision between the particle and the bubble surface, are derived. It is shown that the critical impact parameter is independent of flow characteristics within fairly large terms. The region where a collision between particle and bubble is improbable and, consequently, the flotation process ineffective, is indicated.

A perfect flow around the particle was assumed in [1] when solving a similar problem without taking into consideration inertial factors. In [2] direct hydrodynamic interaction between particle and bubble surface through a thin interlayer of liquid was considered instead of convergence of the particle and bubble surface.

1. The equations of motion of the particle and the generating approximation of the problem. We represent the equation of motion of a small spherical particle in a liquid in dimensionless variables in conformity with [3] in the form

$$K\left(1+\frac{\varkappa}{2}\right)\frac{d\eta}{d\tau}=\frac{3}{2}K\varkappa\frac{d\zeta}{d\tau}-G\mathbf{j}-(\eta-\zeta)- \tag{1.1}$$

$$3\sqrt{\frac{K\varkappa}{2\pi}}\int\limits_{-\infty}^{\tau}\left(\frac{d\eta}{d\tau}-\frac{d\zeta}{d\tau}\right)_{\tau=\tau'}\frac{d\tau'}{\sqrt{\tau-\tau'}}+\frac{3}{2}\frac{K\varkappa}{R}\frac{d}{d\rho}\frac{d\zeta}{d\rho}$$

$$\eta=\frac{d\rho}{d\tau}$$

$$\eta=\mathbf{v}/V,\quad \zeta=\mathbf{u}/V,\quad \tau=Vt/L,\quad \rho=\mathbf{r}/L=\tau_\rho/\varepsilon$$

$$\varkappa=d_1/d_2,\quad G={}^2/_9(d_2-d_1)ga^2/(\mu V)$$

$$K={}^2/_9\,Va^2d_2/(L\mu),\quad R=2LVd_1/\mu$$

where a and d_2 are the characteristic dimension and density of the particle; d_1 and μ are the density and dynamic viscosity of the liquid; g is the acceleration of gravity; $\mathbf{j}$ is a unit vector directed vertically upward; $\mathbf{r}$ is the radius vector of the particle center; $\mathbf{v}(t)$ and $\mathbf{u}(\mathbf{r}, t)$ are the velocity vectors of the particle and liquid, respectively, L and V are the characteristic linear and velocity units of the flow; K and R are the Stokes and Reynolds numbers; $\partial / \partial \mathbf{r}$ is the Hamiltonian (operator), and $\partial \mathbf{u} / \partial \mathbf{r}$ is a 3×3 matrix. The derivative $d\mathbf{u} / dt = \partial \mathbf{u} / \partial t + \mathbf{v} \partial \mathbf{u} / \partial \mathbf{r}$ of velocity of a particle of liquid and the acceleration of the solid particle $d\mathbf{v} / dt$ in the integral in (1.1) are calculated at instant $t = t'$ and $\mathbf{r} = \mathbf{r}(t')$.

Let us assume that the liquid flow is steady and axisymmetric, and that its velocity at infinity is V. In the spherical system of coordinates (r, φ) attached to some point of the axis of symmetry the stream function $\Psi(r, \varphi)$ is of the form

$$r \to \infty, \quad \Psi = -{}^1/{}_2 V r^2 \sin^2 \varphi \tag{1.2}$$

The radial ζ_ρ and tangential ζ_φ velocity components of the liquid are determined by the relations

$$\zeta_\rho = \frac{\varepsilon^2}{\sin \varphi} \frac{\partial \psi}{\partial \varphi}, \quad \zeta_\varphi = \frac{\varepsilon^3}{\sin \varphi} \frac{\partial \psi}{\partial \varepsilon}, \quad \psi = \frac{\Psi}{VL^2} \quad \left(\varepsilon = \frac{L}{r}\right)$$

Formulas (1.1) are equivalent to four integro-differential equations in the unknown $\varepsilon, \varphi, \eta_\rho = \boldsymbol{\eta}\boldsymbol{\tau}_\rho$ and $\eta_\rho = \boldsymbol{\eta}\boldsymbol{\tau}_\varphi$. We reduce the order of that system by projecting the second of Eqs. (1.1) in the radial direction, which then assume the form

$$\eta_\rho \varepsilon d\varphi / d\varepsilon + \eta_\varphi = 0$$

In the case of low Stokes numbers $(K \ll 1)$ the solution is sought in the form

$$\boldsymbol{\eta} = \sum_{n=0}^{\infty} K^{n/2} \boldsymbol{\eta}_n, \quad \varphi = \sum_{n=0}^{\infty} K^{n/2} \varphi_n \tag{1.3}$$

In the generating approximation $(K = 0)$ we have

$$\boldsymbol{\eta}_0 - \boldsymbol{\zeta}_0 + G\mathbf{j} = 0, \quad \eta_{\rho 0} \varepsilon d\varphi_0 / d\varepsilon + \eta_{\varphi 0} = 0 \tag{1.4}$$
$$\boldsymbol{\zeta}_0 = \boldsymbol{\zeta}|_{\varphi = \varphi_0}, \quad \eta_{\rho 0} = \boldsymbol{\eta}_0 \boldsymbol{\tau}_{\rho 0}, \quad \eta_{\varphi 0} = \boldsymbol{\eta}_0 \boldsymbol{\tau}_{\varphi 0}$$

where $\boldsymbol{\tau}_{\rho 0}$ and $\boldsymbol{\tau}_{\varphi 0}$ are basis vectors of the particle generating trajectory. In the generating approximation only the Archimedean force and the viscous resistance to Stokes motion are taken into account. The problem of perfect flow around a spherical bubble was solved in [1] in this approximation. Substitution of expressions for $\eta_{\rho 0}$ and $\eta_{\varphi 0}$ obtained from the first vector equation in (1.4) into the second yields for the particle generating trajectory the equation

$$\frac{G}{2} \frac{\sin^2 \varphi_0}{\varepsilon^2} - \psi_0 = \frac{G+1}{2} \xi^2, \quad \psi_0 = \psi(\varepsilon, \varphi_0) \tag{1.5}$$

where ξ is the constant of integration. Passing in (1.5) to limit $\varepsilon \to 0$, we find on the strength of (1.2) that the constant ξ is the dimensionless impact parameter (the distance between the rectilinear trajectory of the particle and the vertical axis of symmetry normalized with respect to the characteristic dimension L).

To obtain the general solution of variational equations of the generating system (1.4) it is sufficient, according to Poincaré's theorem [4], to determine the derivative $\partial \varphi_0 / \partial \xi$. We have

$$\delta\varphi = A\varepsilon^2 / (G\,\omega_0 \sin\varphi_0 - \varepsilon^2 \partial\psi_0 / \partial\varphi_0), \quad A = \text{const} \tag{1.6}$$

In conformity with (1.4) the asymptotic behavior of variation $\delta\varphi$, as $\varepsilon \to 0$ is defined by

$$\delta\varphi = A\varepsilon / (\xi\sqrt{G+1})$$

For the determination of successive approximations $\varphi_1, \varphi_2, \ldots$ we have inhomogeneous linear differential equations of the first order whose homogeneous parts are the same as in the variational equations of the generating system. Hence their general solutions are of the form of a sum of particular solutions that corresponds to the right-hand side and of the solution of the homogeneous equation that coincides with (1.6). We assume that: 1) constants in the homogeneous parts of general solutions (such as A in (1.6)) vanish; 2) the particular solutions that correspond to right-hand sides decrease in proportion to ε^α, where $\alpha \geqslant 2$, as $\varepsilon \to 0$. The latter can always be achieved by adding to the particular solution a term of the form (1.6). Taking additionally into account homogeneous solutions of the type (1.6) is equivalent to the substitution of expansion $\xi + \sqrt{K}\xi_1 + O(K)$ for the constant ξ in the integral (1.5). Consequently, since $\partial\varphi_i / \partial\varepsilon \underset{\varepsilon\to 0}{\longrightarrow} 0$, ξ is the true (not only the generating) impact parameter

$$\xi = \lim_{\varepsilon\to 0} (\varepsilon^{-1} \sin\varphi)$$

Note that the small parameter K in the input system (1.1) is the coefficient at the higher derivative $d\eta / d\tau$ and that the order of the generating system (1.4) is accordingly lower by two orders than that of the input one. Hence only a particular solution of (1.1) can be constructed in the form of series (1.3), which would depend on the single constant ξ and correspond to uniform settling of the particle at infinity.

We shall now determine the coefficients of expansion (1.3). Differentiating the first of Eqs. (1.4) with respect to τ, and taking into account that $d\mathbf{j} / d\tau = 0$, we obtain

$$d\eta_0 / d\tau - \eta_0 (d\zeta / d\rho)_0 = 0 \tag{1.7}$$

where parentheses indicate that the components of matrix $d\zeta / d\rho$ are calculated in the generating approximation. Equating the coefficients at $\sqrt{K}$ in (1.1) and allowing for (1.7), we obtain for the unknown η_1 and φ_1 a homogeneous system in the form of variational equations of the generating system. This means that $\eta_1 = \varphi_1 = 0$ and the integral term in the first of Eqs. (1.1) affect only the approximation of the order of $O(K^{3/2})$.

2. Derivation of the second and third approximations. Let us determine the projections of vectors η_i $(i = 2,3)$ on the generating polar directions $\tau_{\rho 0}$ and $\tau_{\varphi 0}$. Since the directions $\tau_{\rho 0}$ and $\tau_{\varphi 0}$ of the generating and τ_ρ and τ_φ of the true polar basis vectors differ by the small angle $\varphi - \varphi_0 = K\varphi_2 + O(K^{3/2})$, hence

$$\tau_\rho = \tau_{\rho 0} + \varphi_2 \tau_{\varphi 0} K + \varphi_3 \tau_{\varphi 0} K^{3/2} + O(K^2)$$

$$\tau_\varphi = \tau_{\varphi 0} - \varphi_2 \tau_{\rho 0} K - \varphi_3 \tau_{\rho 0} K^{3/2} + O(K^2)$$

The particle radial and tangential velocity components are represented in the form of expansions

$$\eta_\rho = \eta_{\rho 0} + (\eta_{\rho 2} + \varphi_2 \eta_{\varphi 0}) K + (\eta_{\rho 3} + \varphi_3 \eta_{\varphi 0}) K^{3/2} + O(K^2) \qquad (2.1)$$
$$\eta_\varphi = \eta_{\varphi 0} + (\eta_{\varphi 2} - \varphi_2 \eta_{\rho 0}) K + (\eta_{\varphi 3} - \varphi_3 \eta_{\rho 0}) K^{3/2} + O(K^2)$$

Note that the generating basis vectors $\tau_{\rho 0}$ and $\tau_{\varphi 0}$ are independent of the Stokes number K.

Assuming that $\varkappa / R = O(K)$ and taking into account (2.1), we obtain for the determination of approximations of order $O(K)$ equations of the form

$$\eta_{\rho 2} = \lambda \left(\frac{d\eta_{\rho 0}}{d\varepsilon} + \frac{\eta_{\varphi 0}^2}{\varepsilon \eta_{\rho 0}} \right) + \varphi_2 \left(\frac{\partial \eta_{\rho 0}}{\partial \varphi_0} - \eta_{\varphi 0} \right) \qquad (2.2)$$
$$\eta_{\varphi 2} = \lambda \left(\frac{d\eta_{\varphi 0}}{d\varepsilon} - \frac{\eta_{\varphi 0}}{\varepsilon} \right) + \varphi_2 \left(\frac{\partial \eta_{\varphi 0}}{\partial \varphi_0} + \eta_{\rho 0} \right)$$
$$\varepsilon \frac{d\varphi_2}{d\varepsilon} + \frac{\partial}{\partial \varphi_0} \left(\frac{\eta_{\varphi 0}}{\eta_{\rho 0}} \right) \varphi_2 = \lambda \left[\frac{d}{d\varepsilon} \left(\frac{\eta_{\varphi 0}}{\eta_{\rho 0}} \right) - \frac{\eta_{\varphi 0}}{\varepsilon \eta_{\rho 0}} \left(1 + \frac{\eta_{\varphi 0}^2}{\eta_{\rho 0}^2} \right) \right]$$
$$\lambda = (1 - \varkappa) \eta_{\rho 0} \varepsilon^2, \qquad \frac{d}{d\varepsilon} = \frac{\partial}{\partial \varepsilon} + \frac{d\varphi_0}{d\varepsilon} \frac{\partial}{\partial \varphi_0}$$

where the correction φ_2 is the particular solution of the last of Eqs. (2.2) that satisfies the condition $\varphi_2 / \varepsilon \xrightarrow[\varepsilon \to 0]{} 0$. By virtue of (1.6) the solution reduces to the form

$$\varphi_2 = -(1 - \varkappa) F_1(\varepsilon, \xi) / (\eta_{\rho 0} \sin \varphi_0) \qquad (2.3)$$
$$F_1 = \int_0^\varepsilon \eta_{\rho 0}^2 \sin \varphi_0 \left[\varepsilon \left(\frac{d\varphi_0}{d\varepsilon} \right)^3 - \frac{d^2 \varphi_0}{d\varepsilon^2} \right] d\varepsilon$$

Subsequent approximations of the order of $O(K^{3/2})$ are determined by the system

$$\boldsymbol{\eta}_3 = \varphi_3 \boldsymbol{\sigma} + 3 \sqrt{\varkappa / 2\pi} (1 - \varkappa) \mathbf{A} \qquad (2.4)$$
$$\sigma_\rho = \partial \eta_{\rho 0} / \partial \varphi_0 - \eta_{\varphi 0}, \qquad \sigma_\varphi = \partial \eta_{\varphi 0} / \partial \varphi_0 + \eta_{\rho 0}$$
$$\varepsilon \frac{d\varphi_3}{d\varepsilon} + \frac{\partial}{\partial \varphi_0} \left(\frac{\eta_{\varphi 0}}{\eta_{\rho 0}} \right) \varphi_3 = 3 \sqrt{\frac{\varkappa}{2\pi}} (1 - \varkappa) \frac{1}{\eta_{\rho 0}^2} (\boldsymbol{\eta}_0 \times \mathbf{A}) \mathbf{k}$$
$$\mathbf{A} = \int_{-\infty}^{\tau} \left(\frac{d^2 \boldsymbol{\eta}_0}{d\tau^2} \right)_{\tau = \tau'} \frac{d\tau'}{\sqrt{\tau - \tau'}}$$

where $\mathbf{k}$ is the unit vector normal to the plane of motion of the particle. After integration of the last of Eqs. (2.4) we obtain

$$\varphi_3 = 3 \sqrt{\varkappa / 2\pi} (1 - \varkappa) \varepsilon^2 F_2(\varepsilon, \xi) / (\eta_{\rho 0} \sin \varphi_0) \qquad (2.5)$$
$$F_2 = -\int_0^\varepsilon \frac{\sin \varphi_0}{\eta_{\rho 0} \varepsilon^4} \left\{ \boldsymbol{\eta}_0 \times \int_0^\varepsilon \frac{d}{d\varepsilon'} \left(\varepsilon'^2 \eta_{\rho 0}' \frac{d\boldsymbol{\eta}_0'}{d\varepsilon'} \right) \frac{d\varepsilon'}{\Lambda} \right\} \mathbf{k} \, d\varepsilon$$
$$\Lambda = \left(\int_\varepsilon^{\varepsilon'} \frac{d\varepsilon''}{\varepsilon''^2 \eta_{\rho 0}''} \right)^{1/2}$$

where primes indicate that the respective quantities are calculated for $\varepsilon = \varepsilon'$ ($\varepsilon = \varepsilon''$). It follows from (2.5) that the third correction φ_3 satisfies condition

$\varphi_3 / \varepsilon \underset{\varepsilon \to 0}{\longrightarrow} 0$, hence (2.5) is the sought solution.

Note that the approximation of the order of $O(K)$ defines the effects of particle inertia and of the apparent additional mass. The next following approximation of the order of $O(K^{3/2})$ represents the correction for the unsteadiness of flow around the particle with allowance for the effects of inertia and apparent additional mass. The derived solution is valid for any steady axisymmetric flow that is uniform at infinity.

3. The critical impact parameter. Let us consider the motion of a particle near a spherical bubble floating upward at constant velocity V (Fig. 1). For this we locate the origin of the polar system of coordinates (r, φ) at the bubble center and take the bubble radius as the linear scale L of the flow. Properties of the steady flow of liquid around the bubble are determined by the Reynolds

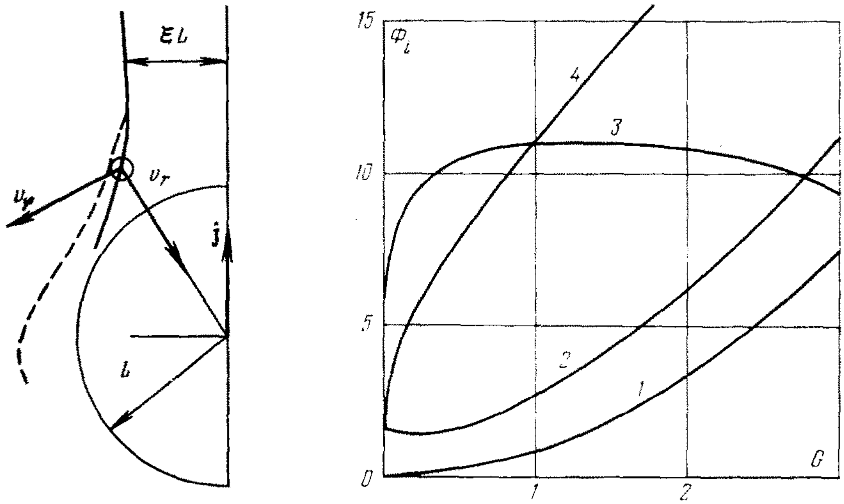

Fig. 1 Fig. 2

number. When $R \gg 1$ the flow is nearly perfect and, in conformity with [5], we have

$$\psi = -\frac{1-\varepsilon^3}{2\varepsilon^2}\sin^2\varphi + \frac{16}{3R}\operatorname{tg}^2\frac{\varphi}{2}(2+\cos\varphi) \times \tag{3.1}$$

$$\left[\frac{1}{2}\operatorname{erfc} z - \frac{1}{2} + \int_0^z z \operatorname{erfc} z\, dz\right], \quad 0 < \varepsilon \leqslant 1$$

$$z = {}^3/_2 \sqrt{R}\,(1-\varepsilon)\,\varepsilon^{-1}\cos^2(\varphi/2) \,/\, \sqrt{2-\cos\varphi}$$

accurate to quantities of higher order of smallness.

If $R \ll 1$ (which in accordance with estimate (2.4) means that the considered particles are heavy in the sense that $\varkappa < O(a/L)$), the flow is close to a Stokes flow, and the following Oseen expansion is valid:

$$\psi = -[2 + \varepsilon + {}^3/_8 R(2 + \varepsilon - (2 + \varepsilon + \varepsilon^2)\cos\varphi) + \tag{3.2}$$
$$O(R^2)] \times {}^1/_4\varepsilon^{-2}(1-\varepsilon)^2\sin^2\varphi, \; O(R) < \varepsilon \leqslant 1$$

The particle critical trajectory osculates the bubble surface, i.e. $\eta_\rho = 0$ when $\varepsilon = 1$. As the distance to the bubble diminishes, the effect of Magnus type forces generated by the particle rotation in the inhomogeneous external flow, increases. Allowance for this effect leads to the appearance in Eq. (1.1) of an additional term of the form $K\,[(d/d\rho)\times\zeta]\times(\eta-\zeta)$. However, owing to the used here generating approximation, that term is of the order of $O(K^2)$. In conformity with (1.5), (2.2), and (2.4) the equation that determines the critical parameter $\xi = \chi$ is of the form

$$\Omega_0 + (1-\varkappa)K\Omega_1 + 3\sqrt{\frac{\varkappa}{2\pi}}\,\frac{1-\varkappa}{G}\,K^{3/2}\Omega_2 + O(K^2) = 0 \qquad (3.3)$$

$$\Omega_0 = 1 - \chi^2(G+1)/G$$

$$\Omega_1 = \Omega_0\left(\frac{\partial^2\varphi_0}{\partial\varepsilon\partial\varphi_0}\right)_{\varepsilon=1}\frac{1}{\chi}\sqrt{\frac{G}{G+1}} - \Omega_1^*$$

$$\Omega_1^* = \Omega_0^{1/2}\left[1 + \frac{(\partial\psi_0/\partial\varepsilon)_{\varepsilon=1}}{(G+1)\chi^2}\right]\left(\frac{\partial\psi_0}{\partial\varepsilon}\right)_{\varepsilon=1} - \frac{F(1,\chi)}{G}$$

$$\Omega_2 = \Omega_0^{1/2}A_\rho(1,\chi) + F_2(1,\chi)$$

Solution of the transcendental equation (3.3) may be represented in the form of series

$$\chi^2 = \frac{1}{1+G}\left[G - (1-\varkappa)\Phi_1 K + 3\sqrt{\frac{\varkappa}{2\pi}}\,(1-\varkappa)\Phi_2 K^{3/2} + O(K^2)\right] \qquad (3.4)$$

$$\Phi_i = F_i(1, \sqrt{G/(G+1)}) \qquad (i = 1, 2)$$

When determining the coefficients of series (3.4) it was taken into consideration that the stream function of flow around the bubble can be represented for any R in the form $\psi = (1-\varepsilon)\,\theta(\varepsilon,\varphi)$, $\theta(1,\varphi) < \infty$.

Because of this the improper part

$$\int_0^1 \frac{f_1(\varepsilon,K)\,d\varepsilon}{\sqrt{1-\varepsilon+Kf_2(\varepsilon K)}} = \int_0^1 \frac{f_1(\varepsilon,0)\,d\varepsilon}{\sqrt{1-\varepsilon}} - 2f_1(1,0)\sqrt{f_2(1,0)K} + O(K)$$

$$f_1 = \varepsilon^{-2}\sigma_\rho\eta_{\varphi 0}^2\sin^2\varphi_0 P^{-1/2} + O(K^{1/2})$$

$$f_2 = (1-\varkappa)G\Phi_1\varepsilon^2P^{-1} + O(K^{1/2})$$

$$P = G^2\left(1 - \varepsilon - \frac{2\varepsilon^2}{G}\theta_0\right) - 2G\varepsilon^2\operatorname{ctg}\varphi_0\frac{\partial\theta_0}{\partial\varphi_0} + \frac{(1-\varepsilon)\varepsilon^4}{\sin^2\varphi_0}\left(\frac{\partial\theta_0}{\partial\varphi_0}\right)^2$$

$$\theta_0 = \theta(\varepsilon,\varphi_0)$$

analytic with respect to $\sqrt{K}$, is separated from the quadrature $F_1(1,\chi)$.

In the first approximation $(K = 0)$ the critical impact parameter is independent of flow properties (the Reynolds number) and of the ratio $\varkappa$ of the particle and liquid densities. The quadratures Φ_1 and Φ_2 are functions of the Reynolds number and, in conformity with (2.3), (2.5), (3.2), and (3.4), are for $R = 0$ and $R = \infty$ of the form

$$\Phi_1 = G\int_0^1 \frac{f - \varepsilon f'/2}{f^2\sqrt{1 - G\varepsilon^2/f}}\,X_1\,d\varepsilon \qquad (3.5)$$

$$X_1 = {}^3/_2\,\varepsilon f f' + f^2 - (2\varepsilon f f' + f^2 - \varepsilon^2 f'^2/4)\,G\varepsilon^2 f$$

$$\Phi_2(G) = \frac{1}{4}\int_0^1 \frac{\mathrm{tg}\,\varphi_0\, d\varepsilon}{f\varepsilon^3} \int_0^{\varepsilon} \frac{dX_2}{d\varepsilon'}\frac{d\varepsilon'}{\Lambda*}$$

$$X_2 = \varepsilon'^2 f' \cos\varphi'\, d(\sigma_1'\sigma_2 - \sigma_1\sigma_2')\,/\,d\varepsilon'$$

$$\Lambda_* = \left(\int_{\varepsilon'}^{\varepsilon} \frac{d\varepsilon''}{f''\varepsilon''^2 \cos\varphi''}\right)^{1/2}$$

$$f|_{R=\infty} = G + 1 - \varepsilon^3, \quad f|_{R=0} = G + (2+\varepsilon)(1-\varepsilon)^2\,/\,2$$

$$\sigma_1(\varepsilon) = \varepsilon f^{\cdot} \sin 2\varphi_0, \quad \sigma_2(\varepsilon) = f - Gf^{\cdot}\varepsilon^3\,/\,f; \quad (^{\cdot}) = d\,/\,d\varepsilon$$

The dependence of $\Phi_i^{(d)} = \Phi_i\,|_{R=\infty}$ and $\Phi_i^{(s)} = \Phi_i\,|_{R=0}\,(i = 1, 2)$ on parameter G is shown in Fig. 2, where curves 1—4 represent, respectively, $\Phi_1^{(s)}$, $\Phi_1^{(d)}$, $\Phi_2^{(d)}$ and $\Phi_2^{(s)}$.

If $R \gg 1$, then formula (3.1) is valid. Substituting in the quadratures Φ_1 and Φ_2 the variables of integration $\varepsilon \to z$ in accordance with (3.1) from (3.5) we obtain

$$\Phi_1 = \Phi_1^{(d)} + 2.65\, R^{-1/4} G^{-1/2} (4G^2 + G - 9) \qquad (3.6)$$

$$\Phi_2 = \Phi_2^{(d)} + R^{-3/4} G^{-1/4} (67.69 + 62.98G + 10.57G^2)$$

which is accurate to quantities of higher order of smallness. Note that expansions (3.6) are not valid when $GR^{1/2} \to 0$.

The present investigation indicates the existence of a "no-flotation zone" when $1 \gg G \gg R^{-3/2}$. Such zone is understood here to be the space of dimensionless numbers G, R, $\varkappa$, and K that define the problem similarity criteria, inside which $\chi \equiv 0$ and, consequently, the space in which collision of the particle with the bubble cannot occur. The dimensions of that zone are determined by the condition that expression (3.4) must be positive.

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